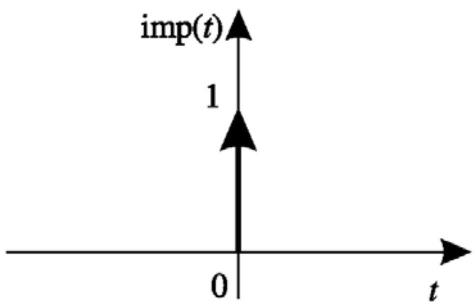
- Impulso di Dirac

$$\mathrm{imp}(t) = 0 \qquad t \neq 0$$

$$\int_{-\infty}^{+\infty} \mathrm{imp}(t)dt = 1$$



$$\mathrm{imp}_\epsilon(t) = \begin{cases} 1/\epsilon & 0 \leq t < \epsilon \\ 0 & \text{altrove} \end{cases}$$

$$\lim_{\epsilon \to 0} \mathrm{imp}_\epsilon(t) = \mathrm{imp}(t)$$

$$\varphi(t)\mathrm{imp}(t-\tau) = \varphi(\tau)\mathrm{imp}(t-\tau)$$

$$\Downarrow$$

$$\int_{-\infty}^{+\infty} \varphi(t)\mathrm{imp}(t-\tau)dt = \varphi(\tau)\int_{-\infty}^{+\infty} \mathrm{imp}(t-\tau)dt = \varphi(\tau)$$

$$\int_{-\infty}^{t} \mathrm{imp}(\tau)d\tau = \mathrm{sca}(t) \qquad t \neq 0$$

$$\frac{d(\mathrm{sca}(t))}{dt} = \mathrm{imp}(t)$$

Risposta all'impulso e movimento forzato

- Risposta all'impulso ($m = 1$, $u(t) = \mathrm{imp}(t)$, $x(0) = 0$)

$$g_x(t) = e^{At}B$$
$$g_y(t) = Ce^{At}B + D\,\mathrm{imp}(t)$$

- ⋆ coincide con il movimento libero prodotto da $x(0) = B$: combinazioni lineari dei modi del sistema
- ⋆ estensione al caso $m > 1$: risposta di x_i (y_i) all'impulso unitario u_j (per $u_k = 0$, $k \neq j$)

- Movimento forzato

$$g_x(t) \star u(t) = \int_{-\infty}^{+\infty} g_x(t-\tau)u(\tau)d\tau = \int_0^t g_x(t-\tau)u(\tau)d\tau$$
$$= \int_0^t e^{A(t-\tau)}Bu(\tau)d\tau = x_f(t)$$

$$g_y(t) \star u(t) = \int_{-\infty}^{+\infty} g_y(t-\tau)u(\tau)d\tau = \int_0^t g_y(t-\tau)u(\tau)d\tau$$
$$= \int_0^t \left(Ce^{A(t-\tau)}B + D\mathrm{imp}(t-\tau)\right)u(\tau)d\tau = y_f(t)$$

$\star$ A diagonalizzabile

$$x_f(t) = T_D^{-1}\hat{x}_f(t) = T_D^{-1}\int_0^t e^{\hat{A}_D(t-\tau)}\hat{B}u(\tau)d\tau$$

$$= T_D^{-1}\int_0^t \text{diag}\left\{e^{s_1(t-\tau)}, e^{s_2(t-\tau)}, \ldots, e^{s_n(t-\tau)}\right\} T_D B u(\tau)d\tau$$

$$y_f(t) = CT_D^{-1}\int_0^t \text{diag}\left\{e^{s_1(t-\tau)}, e^{s_2(t-\tau)}, \ldots, e^{s_n(t-\tau)}\right\} T_D B u(\tau)d\tau$$

$$+ Du(t)$$

- Esempio (precedente)

$$u(t) = \text{sca}(t)$$

  - ⋆ movimento forzato ($s_1 \neq s_2, k \neq 0$)

$$x_f(t) = \begin{bmatrix} 1 & 1 \\ s_1 & s_2 \end{bmatrix} \int_0^t \text{diag}\left\{ e^{s_1(t-\tau)}, e^{s_2(t-\tau)} \right\} d\tau \begin{bmatrix} -\frac{1}{M(s_2-s_1)} \\ \frac{1}{M(s_2-s_1)} \end{bmatrix}$$

$$= \frac{1}{M(s_1-s_2)} \begin{bmatrix} \frac{e^{s_1 t}-1}{s_1} - \frac{e^{s_2 t}-1}{s_2} \\ e^{s_1 t} - e^{s_2 t} \end{bmatrix}$$

$$y_f(t) = \frac{1}{M(s_1-s_2)} \left(\frac{e^{s_1 t}-1}{s_1} - \frac{e^{s_2 t}-1}{s_2} \right)$$

Equilibrio

- $u(t) = \bar{u}$

$$A\bar{x} + B\bar{u} = 0$$
$$\bar{y} = C\bar{x} + D\bar{u}$$

$\star$ A invertibile ($s_i \neq 0$)

$$\bar{x} = -A^{-1}B\bar{u}$$
$$\bar{y} = \left(-CA^{-1}B + D\right)\bar{u}$$

$\star$ $D - CA^{-1}B$: guadagno statico del sistema SISO (rapporto tra uscita e ingresso quando tutte le variabili sono costanti)

- Esempio (precedente)

$$0 = \bar{x}_2$$

$$0 = -\frac{k}{M}\bar{x}_1 - \frac{h}{M}\bar{x}_2 + \frac{1}{M}\bar{u}$$

$$\Downarrow$$

$$\bar{x} = \begin{bmatrix} 1/k \\ 0 \end{bmatrix} \bar{u} \qquad \bar{y} = \frac{1}{k}\bar{u}$$