Chaos and Real World:
Fractal analysis of cardiovascular variability series

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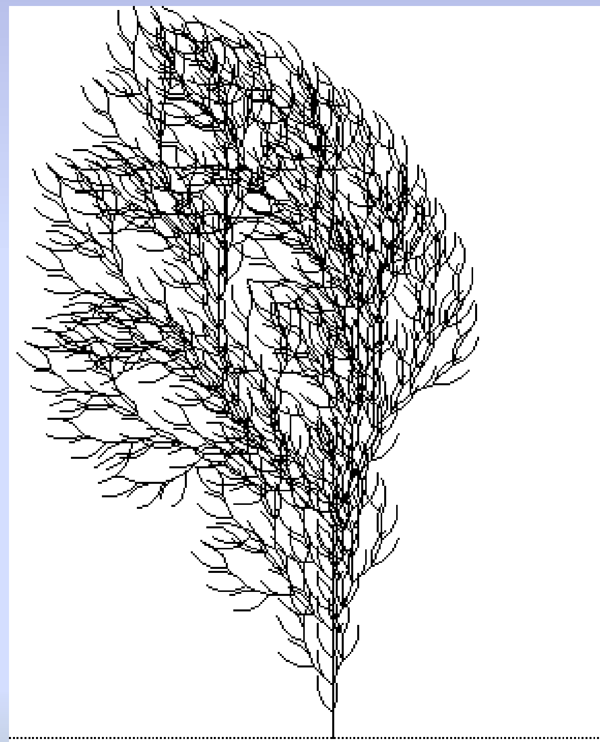
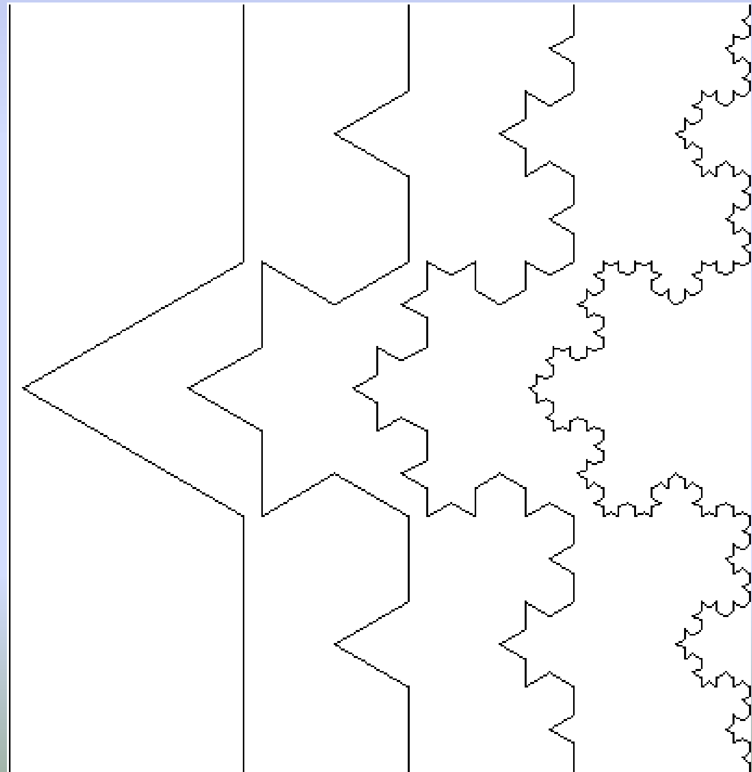
Fractal structures

NATURE

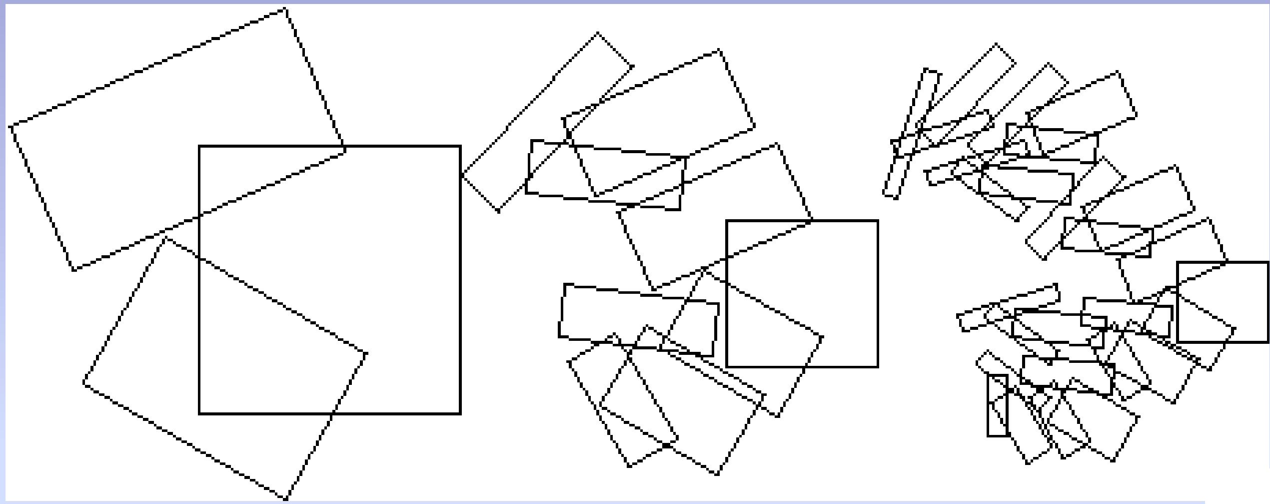
GEOMETRY

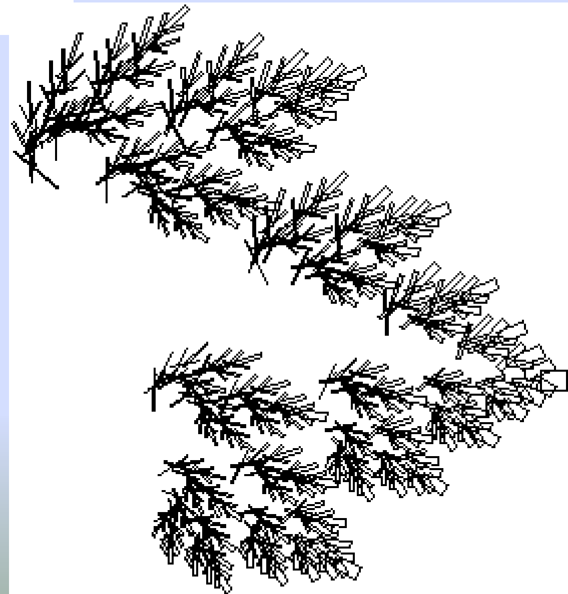


fractal	Euclidean
modern invention	traditional
no specific size or scale	based on a characteristic size or scale
appropriate for geometry in nature	suits description of man made objects
described by an algorithm	described by a usually simple formula

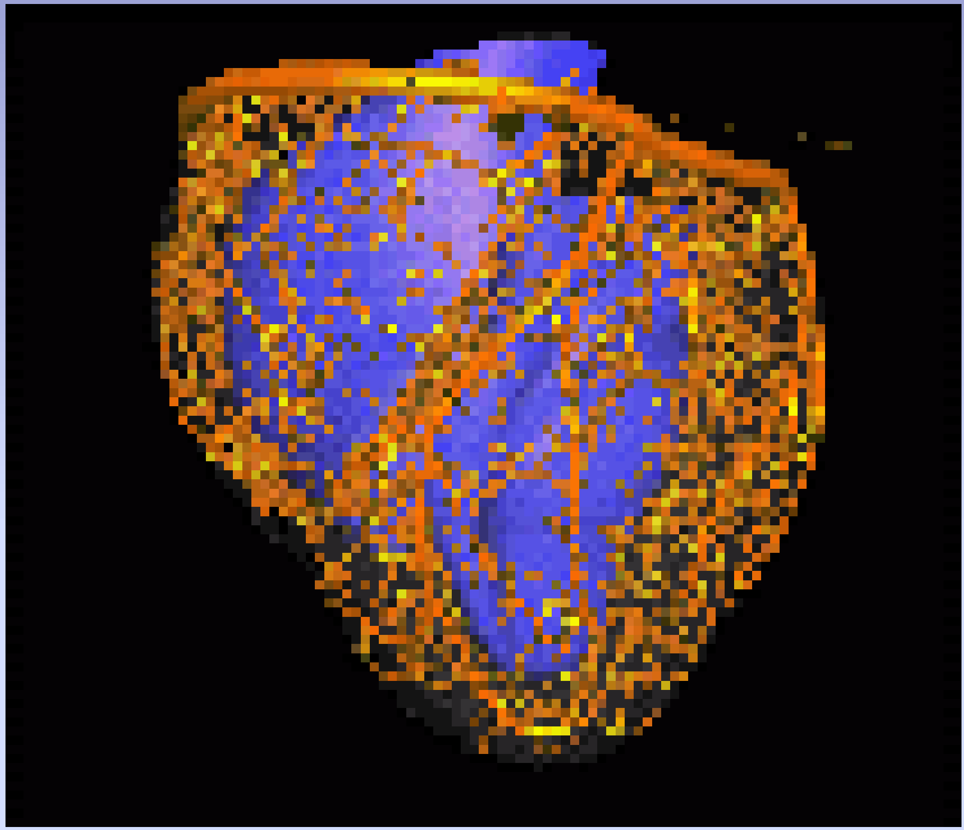


Fractal structure generation



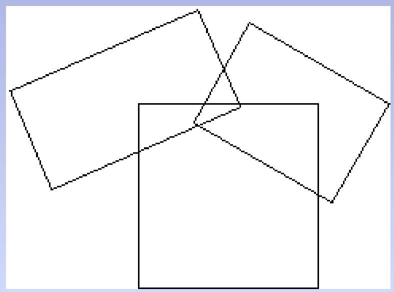
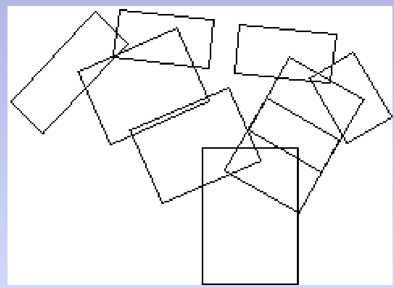
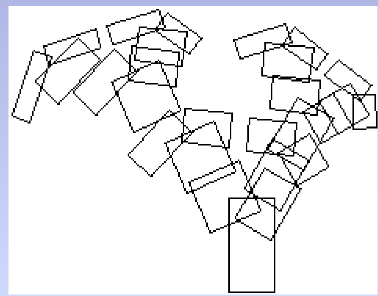
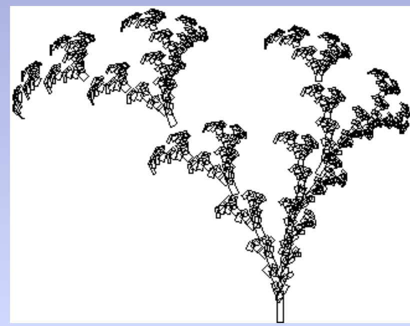


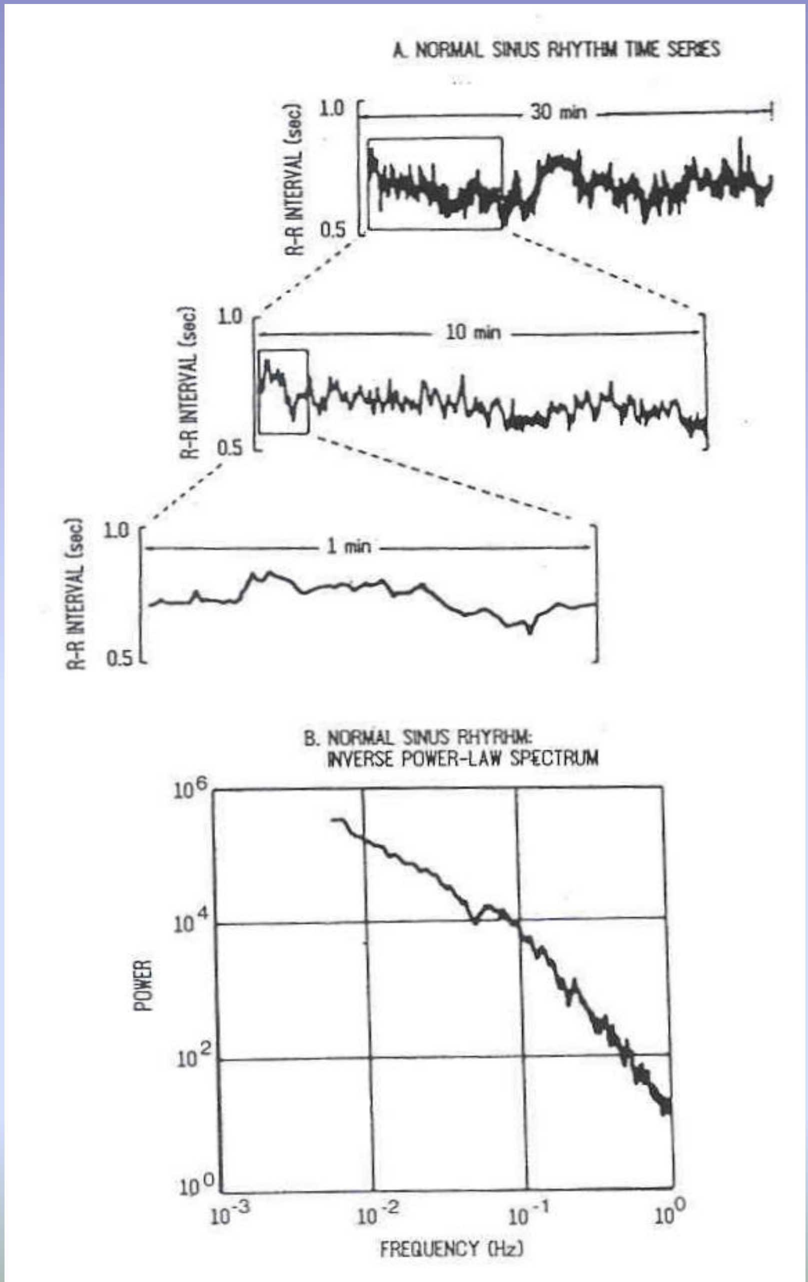
Fractal structures



- Coronary System Model generated through fractal algorithms

Fractal structures



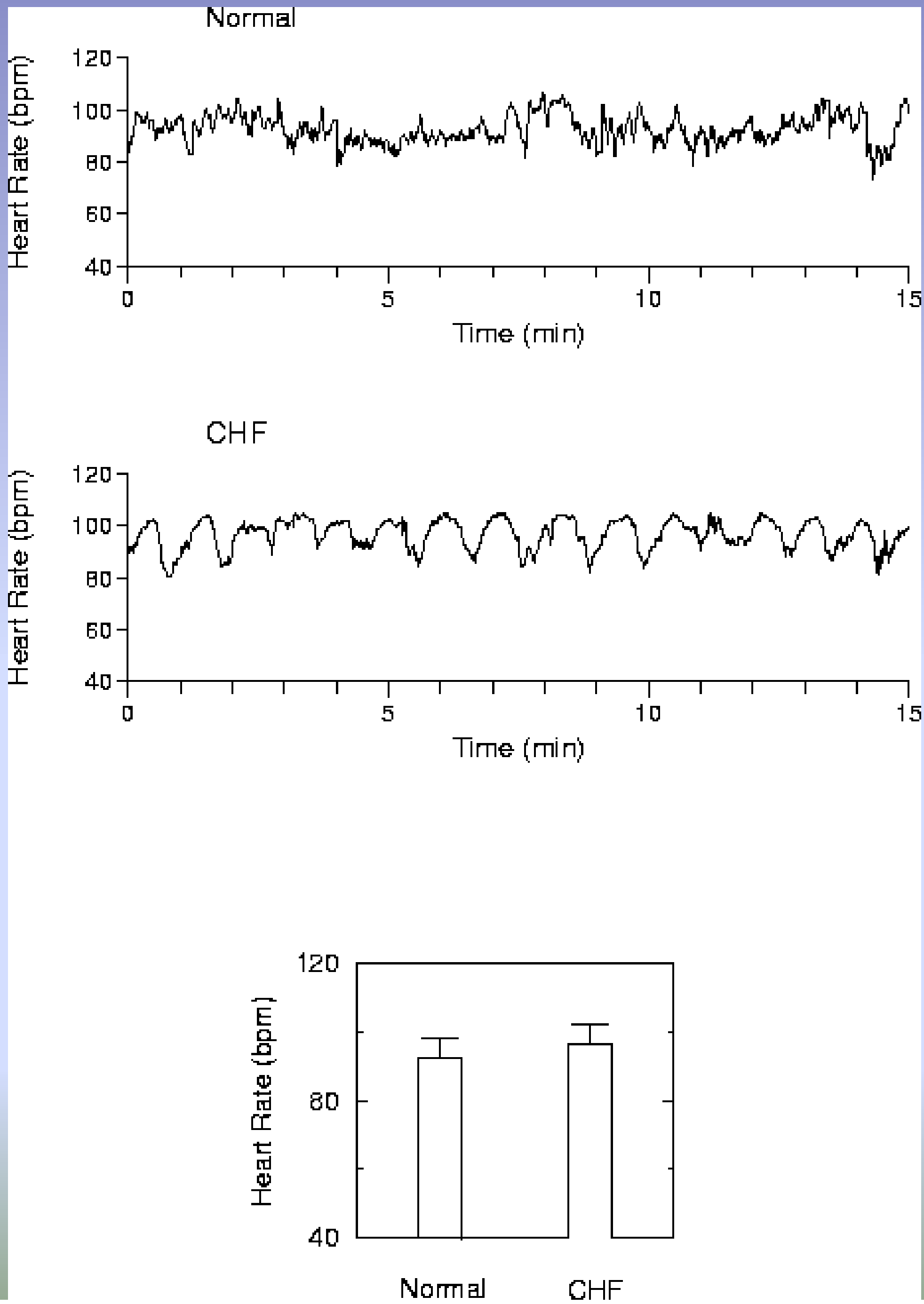
Self-Similarity in the time domain

- Evolution in time of HRV signal shows **SELF-SIMILARITY** properties
- Time series repeats the same pattern at different magnitude degrees.
- It looks similar to a **geometric fractal**

Recent results showed biological signals do not only contains linear harmonic contributions (traditionally identified through spectral analysis techniques) but they possess a fractal like geometry with many rhythmic components interacting over different scales.

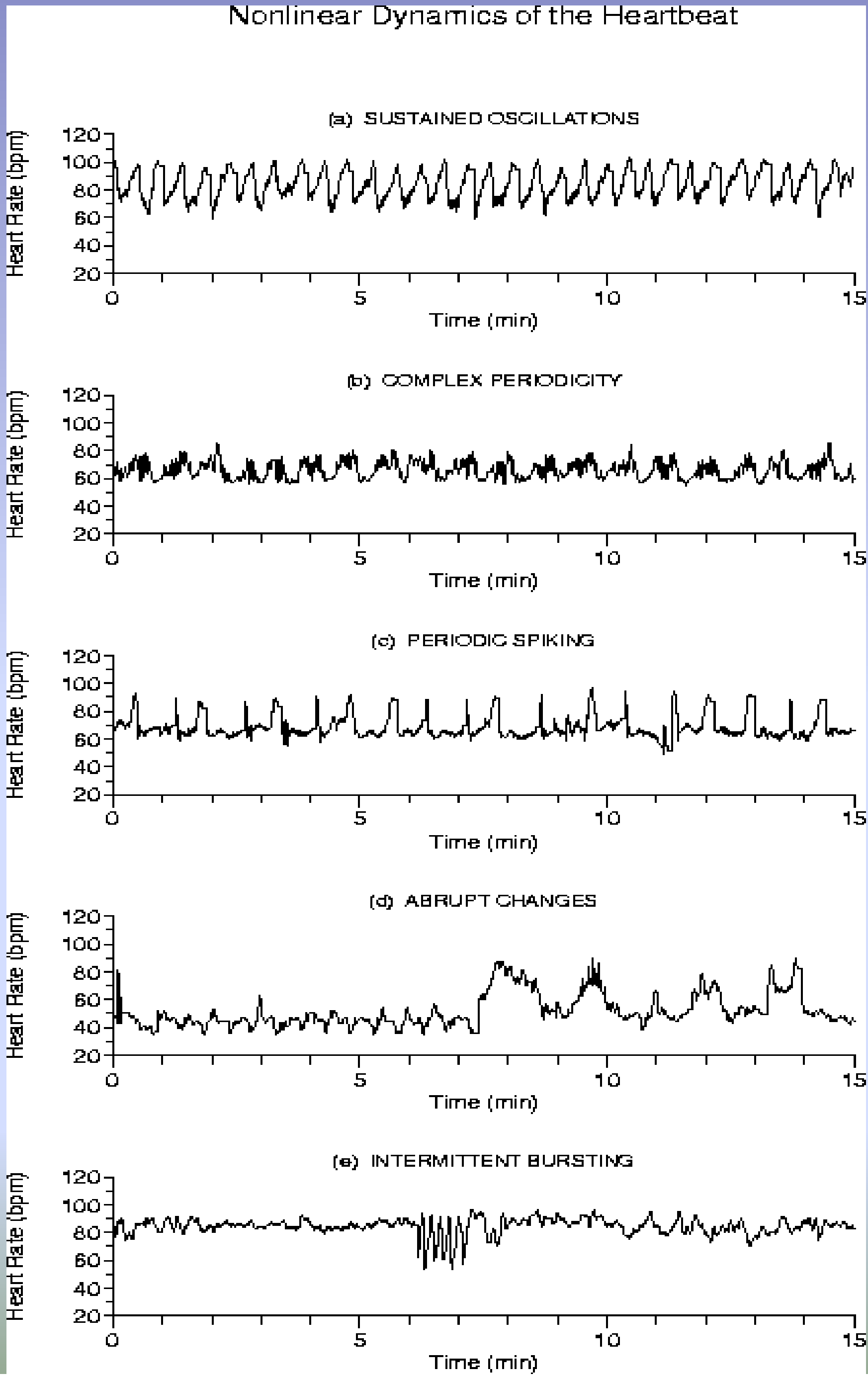
Biological time series can show fractal characteristics in their patterns, as well as in the temporal scales.

Signals with different degrees of magnification of time step, show patterns possessing self-similar characteristics (at a more or less extent).

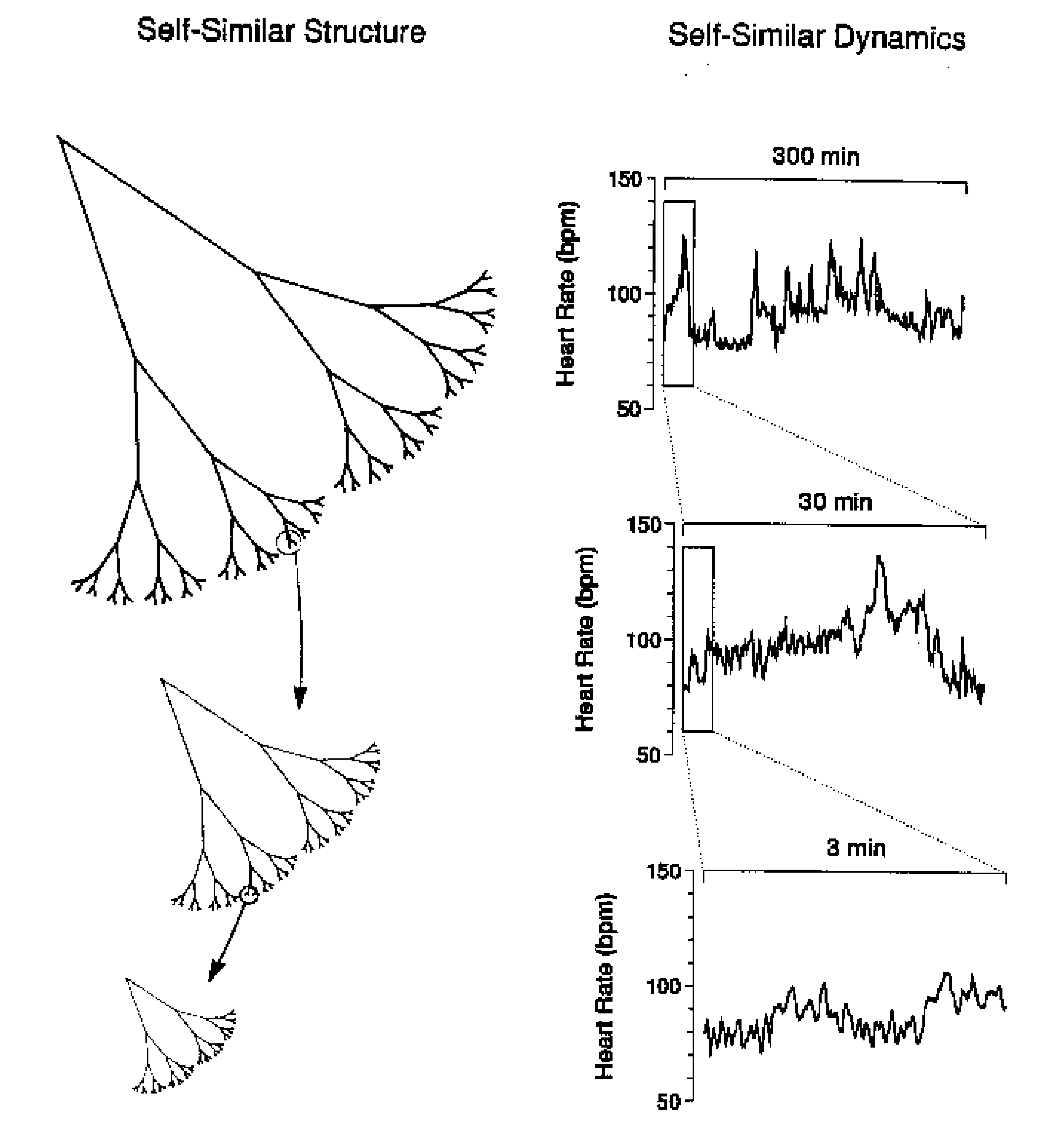


Two heart rate time series, one from a healthy subject (*top*) and the other from a patient with severe congestive heart failure (CHF) (*middle*) have **nearly identical means and variances (*bottom*), yet very different dynamics**.

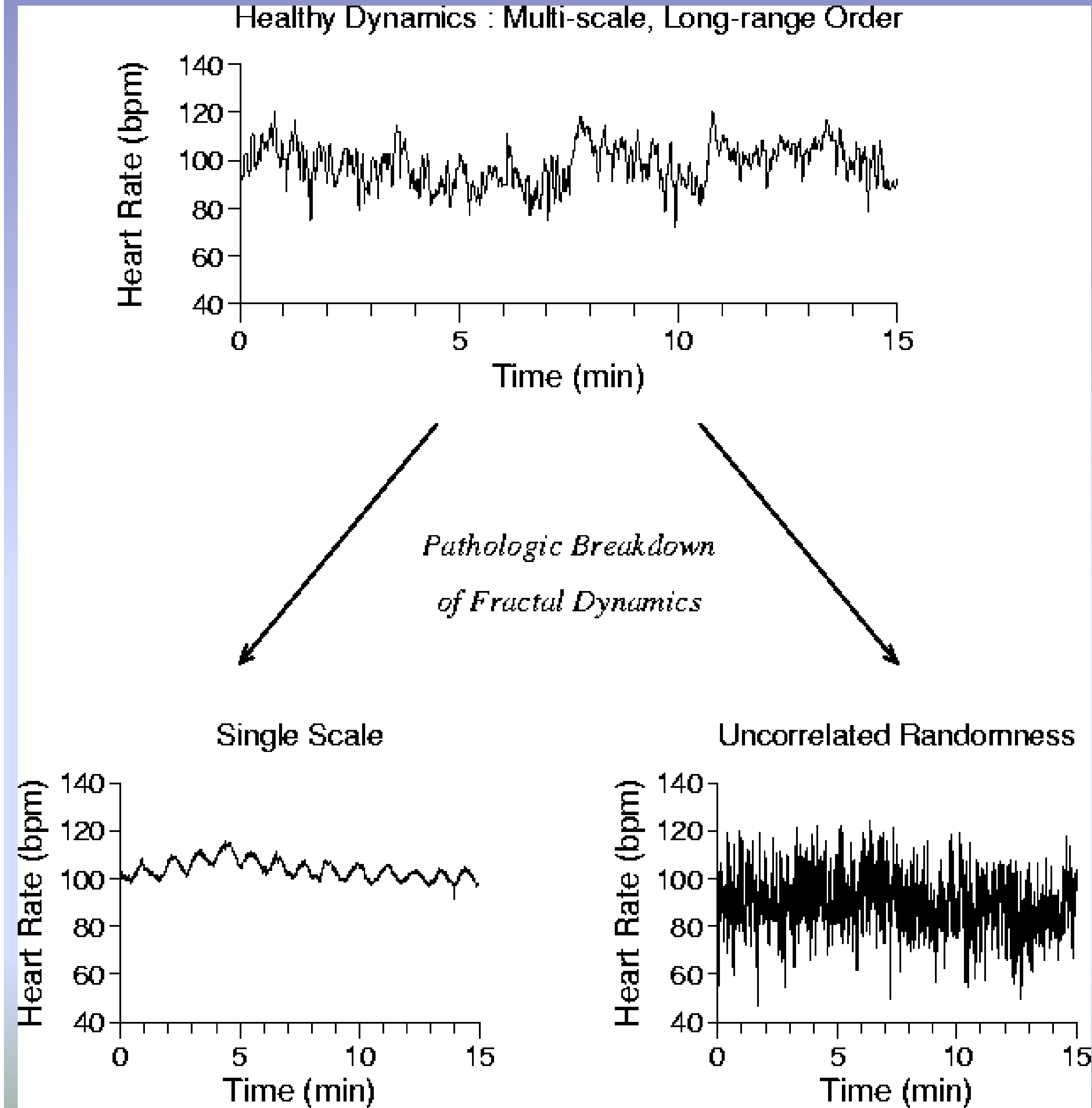
Note that according to classical physiological paradigms based on homeostasis, neuroautonomic control systems should be designed to damp out noise and settle down to a constant equilibrium-like state. However, the healthy heartbeat displays highly complex, apparently unpredictable fluctuations even under steady-state conditions. In contrast, the heart rate pattern from the subject with heart failure shows slow, periodic oscillations that correlate with Cheyne-Stokes breathing.



. Examples of nonlinear dynamics of the heartbeat. Panels (a-c) are from subjects with obstructive sleep apnea syndrome. Panels (d and e) are from healthy subjects at high altitude (~15,000 ft).



Left, schematic of a tree-like fractal has self-similar branchings such that the small scale (magnified) structure resembles the large scale form.
Right, a fractal process such as heart rate regulation generates fluctuations on different time scales (temporal "magnifications") that are statistically self-similar. (Goldberger AL. Non-linear dynamics for clinicians: chaos theory, fractals, and complexity at the bedside. *Lancet* 1996;**347**:1312-1314.)



Breakdown of a fractal physiological control mechanism can lead ultimately either to a highly **periodic output dominated by a single scale or to uncorrelated randomness.**

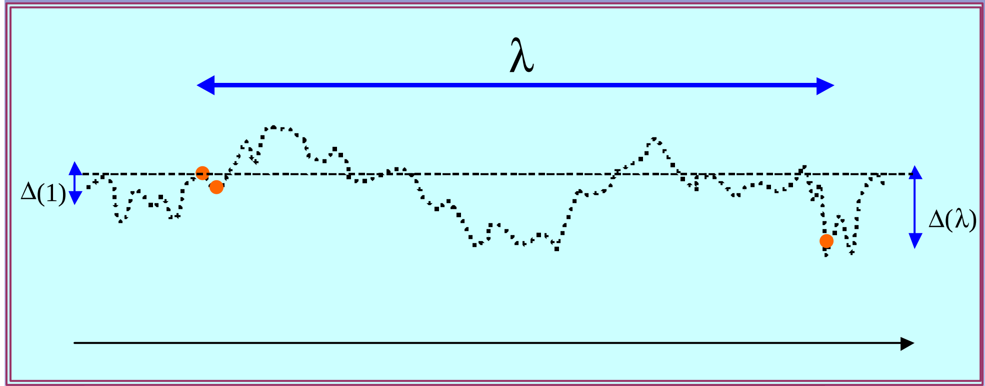
The top heart rate time series is from a healthy subject; bottom left is from a subject with heart failure; and bottom right from a subject with atrial fibrillation. (Goldberger AL. Non-linear dynamics for clinicians: chaos theory, fractals, and complexity at the bedside. *Lancet* 1996;347:1312-1314.)

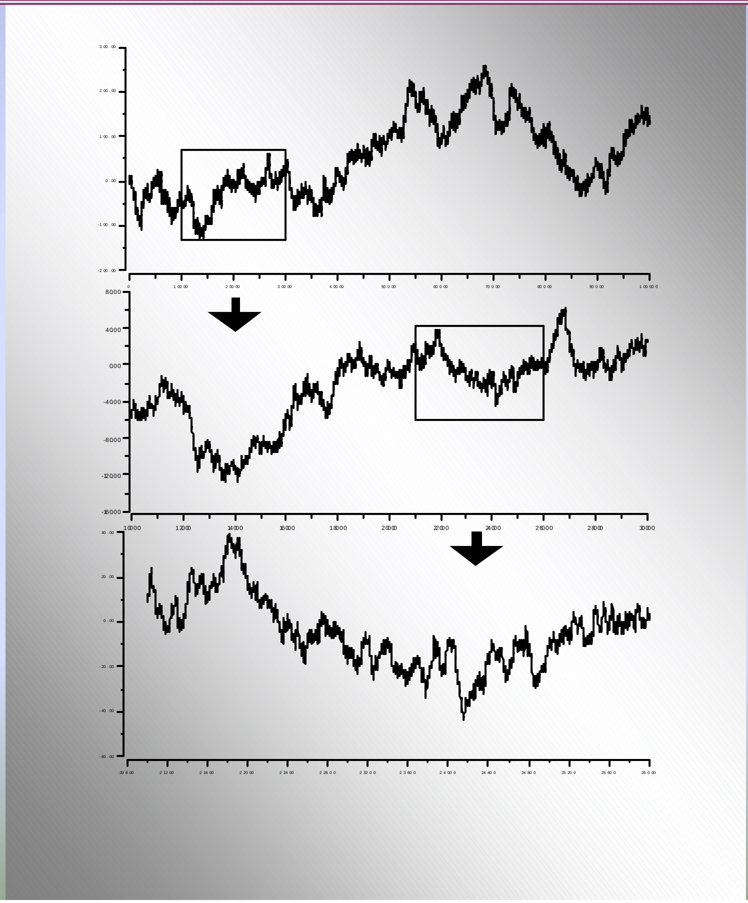
GOAL

- *Proposal of new methods measuring the Self-Similarity H parameter in time series (ex. Heart Rate Variability).*

- *Evaluation of the diagnostic power of the index H in a population with cardiovascular pathologies.*

Self-Similarity: definition





- $x(t) \Rightarrow l^{-H} x(l\, t)$

  (l is the scaling factor)

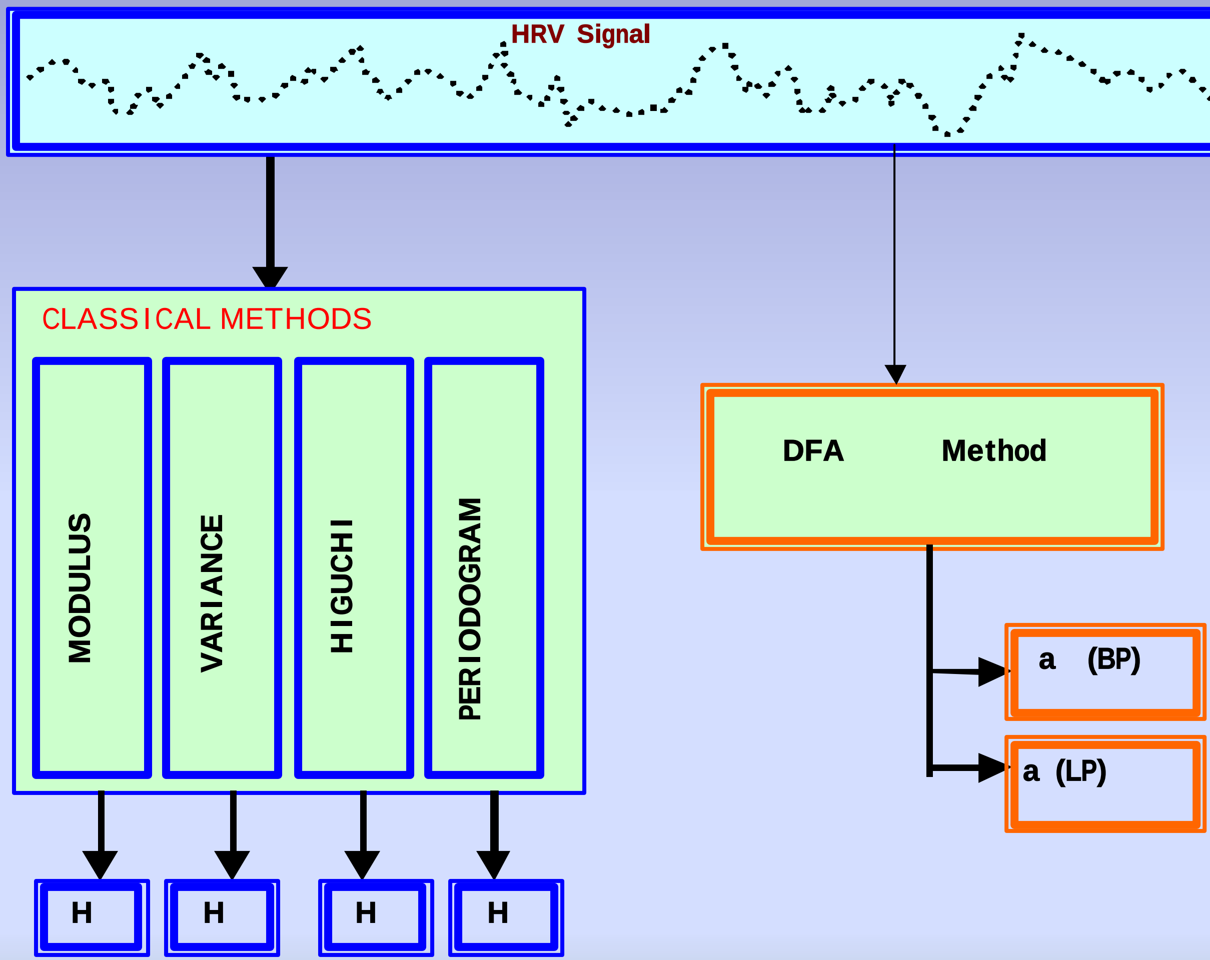
- $x(t) =_d l^{-H} x(l\, t)$

$=_d$ means equality in distributions

- **for fBM:** $\mathbf{D(1) =_d l^{-H} D(l)}$
  - $\mathbf{0 < \mathit{H} < 1}$
- **H is called**
  - self-similarity parameter
  - long period correlation
  - Hurst exponent
  - long period memory

- H ≅ 0 ⇔ negative correlation
- H=0,5 ⇔ uncorrelated signal
- H ≅ 1 ⇔ positive correlation

Methods for the H parameter estimation



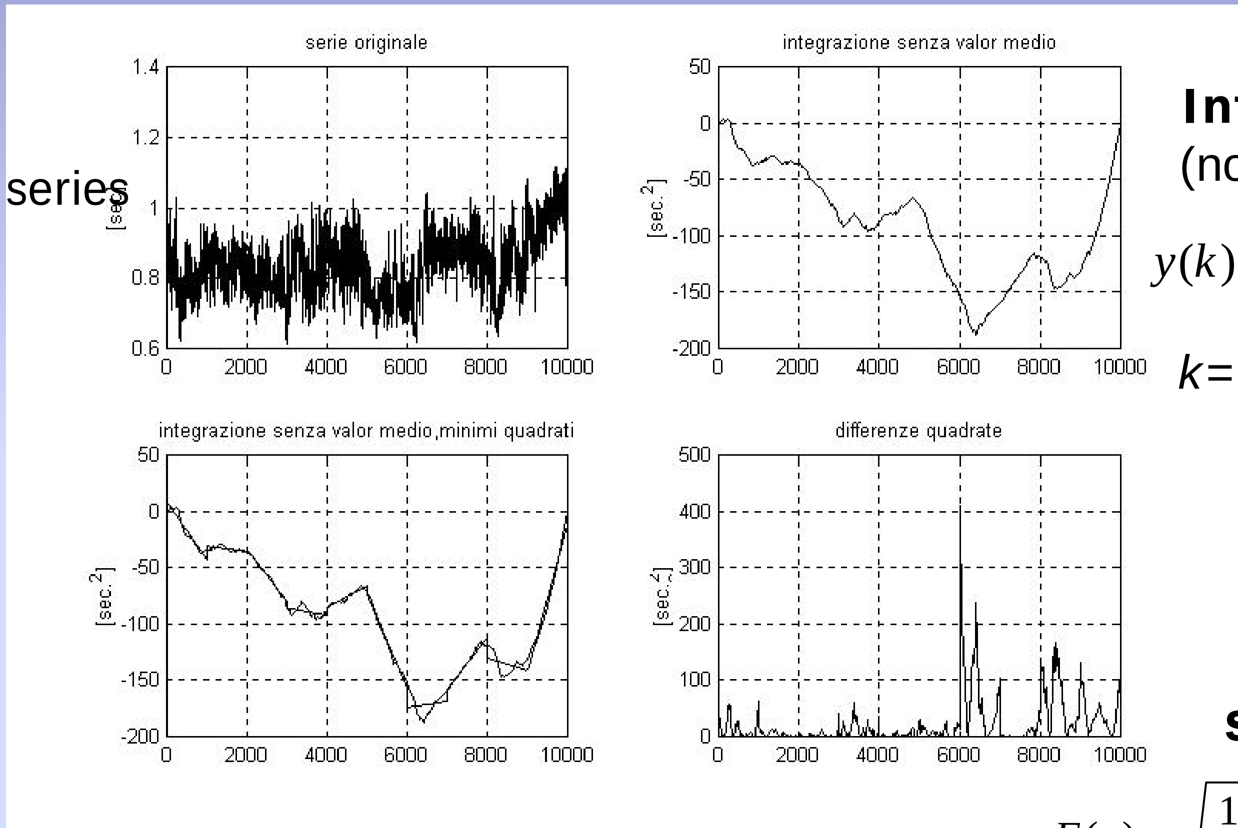
- Absolute values of the aggregated series *(J. Beran, 1994)*
- Variance of the aggregated series *(J. Beran, 1994)*
- Higuchi *(Higuchi, 1988)*
- Periodogram
- Detrended Fluctuation Analysis (DFA) *(C.K. Peng, 1995)*

Methods

Detrended Fluctuations Analysis

1

Original time series



2

Integrated series (no mean value)

$$y(k) = \sum_{i=1}^{k}[x(i) - x_{medio}]$$

$k=1,\ldots,N$

3

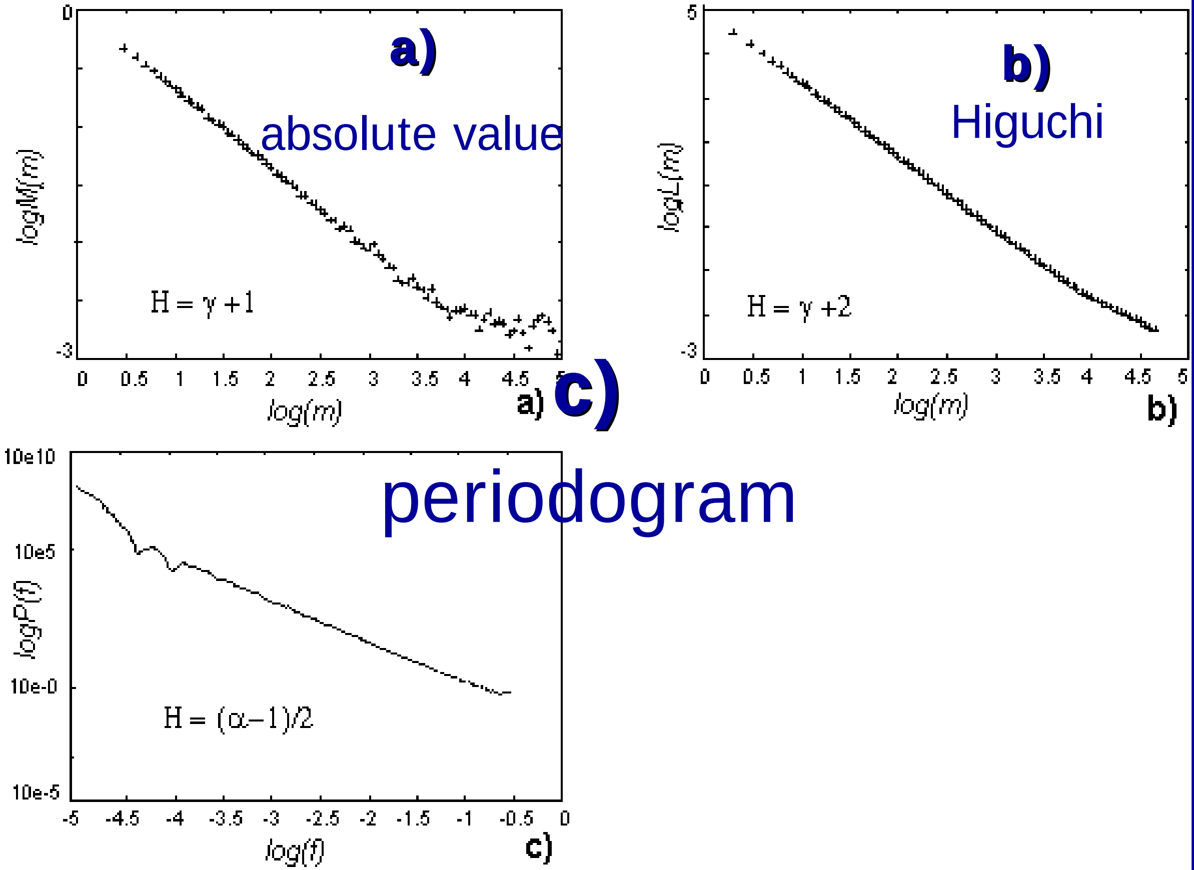
Least squares curve $y_n(k)$, $k=1,\ldots,N$

4

Root mean square error

$$F(n) = \sqrt{\frac{1}{N}\sum_{k=1}^{N}[y(k) - y_n(k)]}$$

Methods



Log-log plots are obtained from a fBm with *H* = 0.3, N=100,000.

EXAMPLE

- time series $x(i)$ ($1 \le i \le N$)
- $X(i) = x(i+1) - x(i)$.
- $X(i)$ is divided into N/m blocks of size m.
- From the average of each block we obtain the aggregated series

$$X^{(m)}(k) = \frac{1}{m} \sum_{i=(k-1)m+1}^{km} X(i)$$

k = 1, 2, ..., N/m .

The *absolute values of the aggregated series* are:

- .

$$M^{(m)} = \frac{1}{N/m} \sum_{k=1}^{N/m} \left| X^{(m)}(k) \right|$$

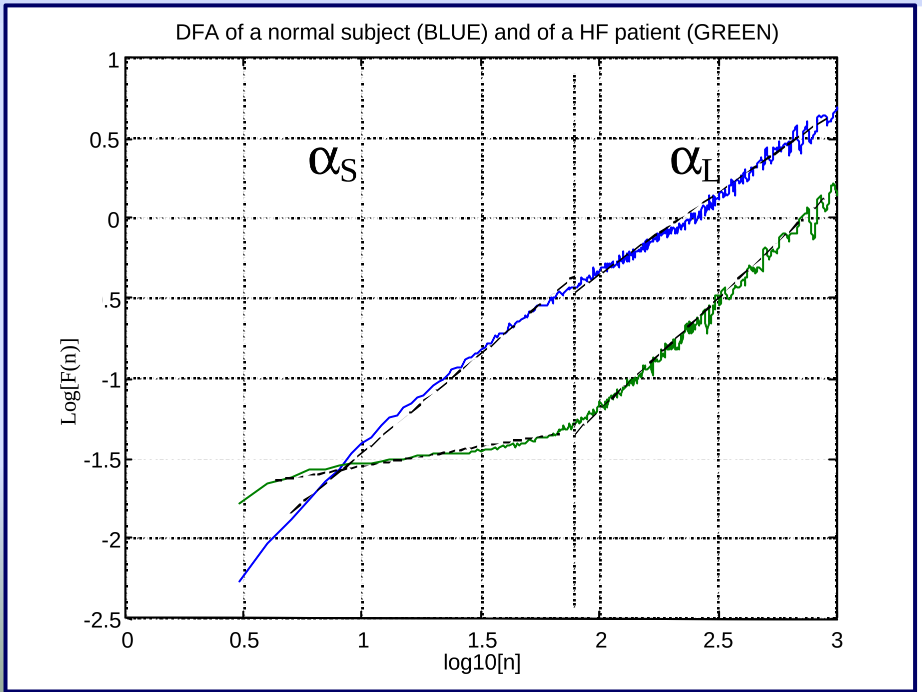
- We repeat this step for different m values (m dimension of the data subset),
- Plot of $M(m)$ vs. m in log-log scale.
- $M(m) \propto m^g$ related to the self-similarity parameter
- $g = H - 1$
- log(M) vs. log(m) plot ↑ straight line with slope g.

results in Heart Failure patients

Detrended Fluctuations Analysis (DFA)

- S: short period (4,000 beats)
- L: long period (10,000 beats)

DFA	Test t
$\alpha_{S,N} > \alpha_{S,HF}$	99.9 %
$\alpha_{L,N} < \alpha_{L,HF}$	99.9 %
$\alpha_{S,N} > \alpha_{L,N}$	99.9 %
$\alpha_{S,HF} < \alpha_{L,HF}$	98 %



Interpretation

NORMALS: Random Walk in short period, 1/f noise in long period.

HF PATIENTS: 1/f noise in both short and long period.

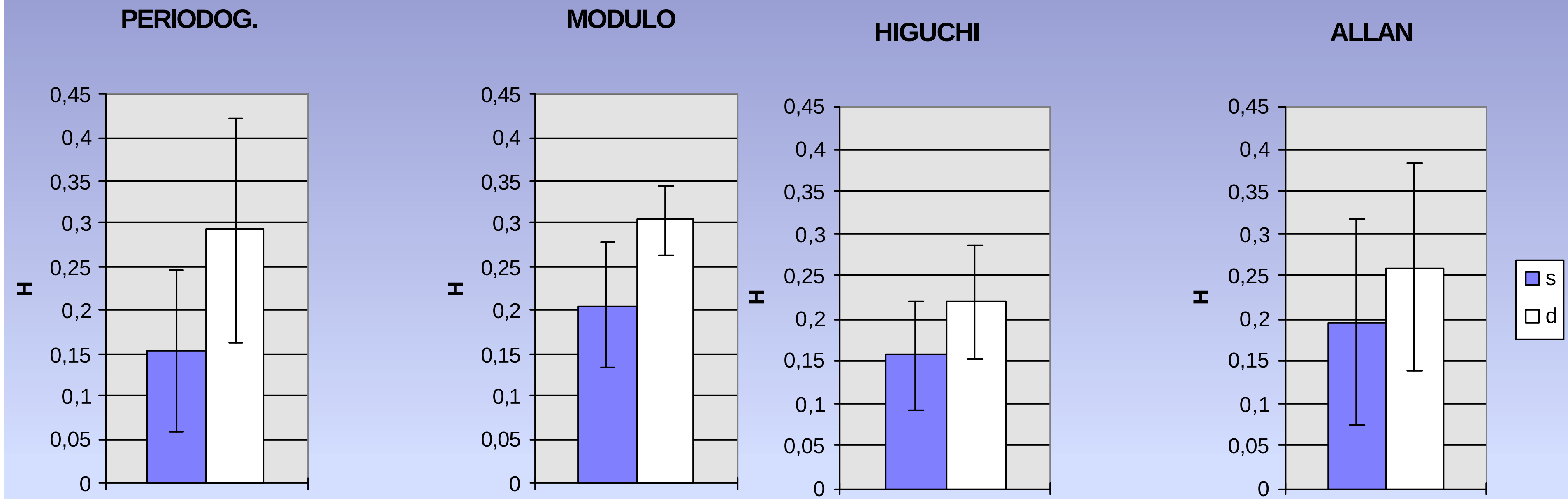
Normal subjects are more correlated than HF patients in long period and less correlated in short period.

results in Heart Failure patients

- ***H*** calculation over a 24 hour HRV signal

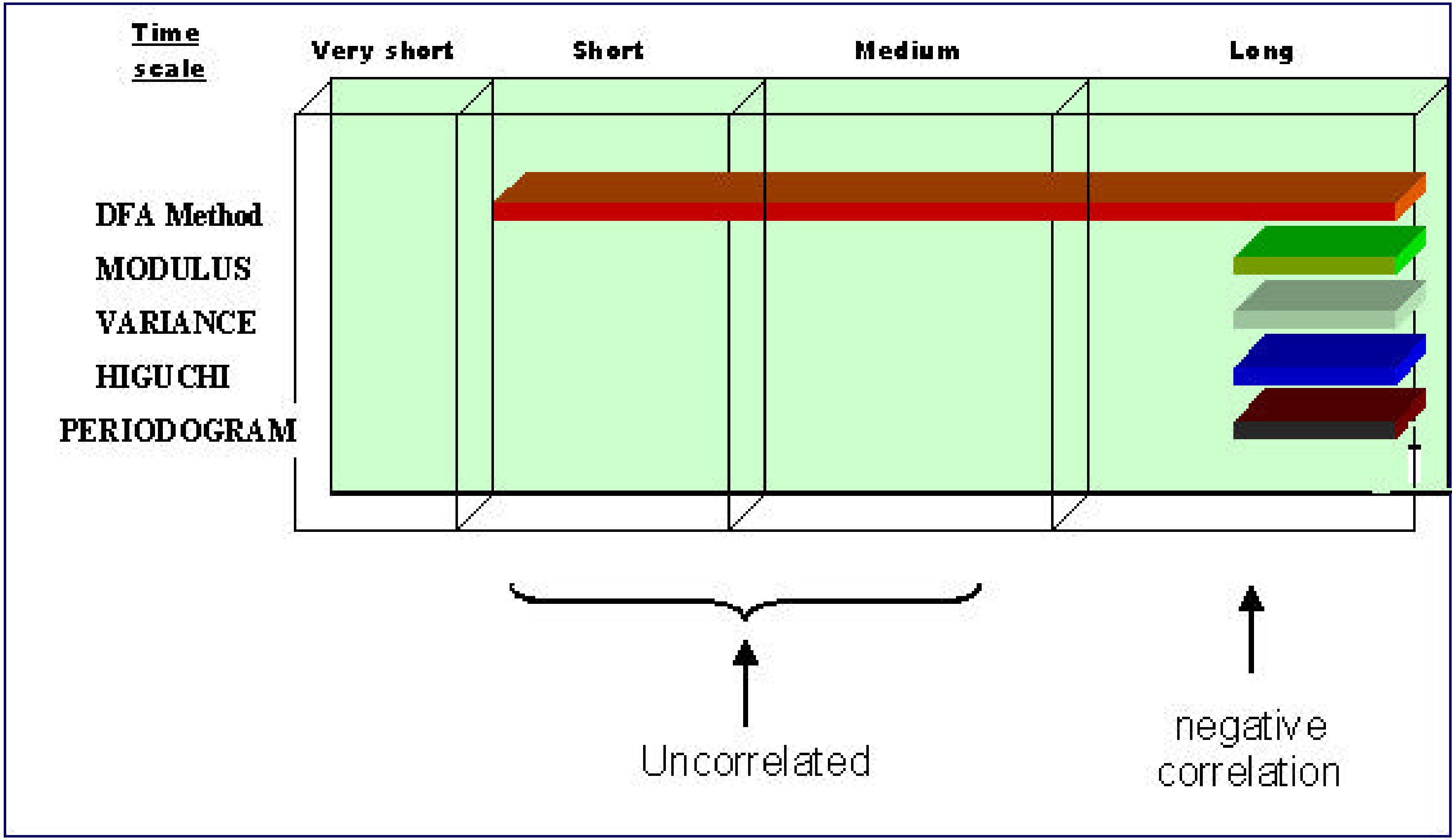
Method H	**Normal** (avg.±std)	**Heart Failure** (avg.±std)	**t-test**
Modulus	0.115.±0.017	0.153.±0.050	p<0.05
Aggr. Var.	0.127.± 0.034	0.205 ±0.072	p<0.05
Higuchi	0.121.± 0.022	0.141.± 0.042	n.s.
Periodogram	0.0925.± 0.054	0.1657.± 0.050	p<0.05

results in ICU patients



p<0.02 *p<0.002* *p<0.03* *p<0.2*

- ➔ Long-term H parameter are significantly different between survived and non-survived subjects.
- ➔ S show α-slope values close to 1 (almost normal values), while D have significantly higher values.
- ➔ The α-slope (periodogram) : 1.44±0.35 (dead) vs 1.13±0.10 (survived)
- ➔ Increase of α values in patients who died ➔➔ prognostic


Time scale
Very short
Short
Medium
Long
DFA Method
MODULUS
VARIANCE
HIGUCHI
PERIODOGRAM
Uncorrelated
negative correlation

Conclusions

- All the presented methods confirm the exponent *H* is a powerful indicator of the neural control activity on the heart over long time scales.
- Further decomposition of *H* value (in short, medium and long time scales) could be used to better understand fractal properties of biological time series.
- Results confirm the presence of fractal and self-similar characteristics in the HRV signal
- Analysis of different groups with various cardiovascular diseases show *H* parameter significantly differentiate healthy vs. pathological subjects.